



Third United Nations  
International Conference  
on the Peaceful Uses  
of Atomic Energy

A/CONF.28/A/632  
CZECHOSLOVAKIA

February 1964

Submitted in:  
ENGLISH and RUSSIAN

Confidential until official release during Conference

Abstract

SOME ITERATIVE METHODS FOR THE CONSTRUCTION OF  
SOLUTIONS OF NEUTRON TRANSPORT EQUATIONS

Ivo Marek

The neutron transport equations can be written in the time independent case in the form

$$(1) \quad Lx = Bx + \lambda Cx + s,$$

where the operators  $L$ ,  $B$ ,  $C$  and the vectors  $x$ ,  $s$  have the following meaning: The operator  $L$  denotes the diffusion and the absorption,  $B$  - the scattering,  $C$  - the fission; the vector  $x$  denotes the density of neutrons or the neutron-flux,  $s$  denotes the exterior sources. Let the equation (1) describe the neutron transport in a given bounded domain  $\mathcal{V}$ . The form of the operators  $L$ ,  $B$ ,  $C$  depends on the approximation in which the neutron transport is concerned (see G.J. Habetler, M.A. Martino, G. Birkhoff, G.I. Marčuk, I. Marek, R.S. Varga, V.S. Vladimirov). In papers of these authors it is proved, beside other results, that the equation

$$(2) \quad Lx = Bx + \lambda Cx$$

has a solution  $x_0$  which is positive in  $\mathcal{V}$ . This solution corresponds to the positive characteristic value  $\lambda_0$ , for which the inequality  $|\lambda| > \lambda_0$  holds for every different characteristic value  $\lambda$  of the equation (2). Moreover this equation has no non-negative solution in  $\mathcal{V}$  other than  $Cx_0$ , where  $C$  is a positive constant. This fact is a consequence of properties of the operators  $(L - B)^{-1}C$ ,  $C(L - B)^{-1}$ . Easily it can be proved that the value  $\mu_0 = \lambda_0^{-1}$  is the dominant eigenvalue of the operator  $T$ , where  $T$  denotes one of the operators  $(L - B)^{-1}C$ ,  $C(L - B)^{-1}$ .

To construct the eigenelements of the equation (2) and the solutions of the equation (1) there are suitable the iterative methods. Our purpose is to present a general iterative schedule and to prove its convergence.

Let  $X$  be a Banach space of vector-functions on  $\mathcal{U}$ . Let  $y'_n, z'_n, y'$  be continuous functionals, for which

$$y'_n(x) \rightarrow y'(x), z'_n(x) \rightarrow y'(x)$$

for every  $x \in X$  and

$$y'_n(\lambda x) = \lambda y'_n(x), z'_n(\lambda x) = \lambda z'_n(x)$$

for  $x \in X$  and  $\lambda > 0$ .

Let the relations  $y'_n(x) \geq 0, z'_n(x) \geq 0$  hold for every non-negative vector  $x \in X$ . Thus  $y'(x) \geq 0$  hold for such vectors. Further let  $y'(x_0) > 0$ , where  $x_0$  is the positive eigenvector of the equation (2).

Let us put

$$x_{(n+1)} = \mu_n^{-1} T x_{(n)},$$

where  $x_{(0)}$  is a positive vector. Further let be

$$\mu_n = \frac{z'_n(T x_{(n)})}{y'_n(x_{(n)})}$$

Theorem 1. The sequences  $\{\mu_n\}, \{x_{(n)}\}$  are convergent:

$$\mu_n \rightarrow \lambda_0^{-1}, x_{(n)} \rightarrow c x_0,$$

where  $L x_0 = B x_0 + \lambda_0 C x_0$  and  $c$  is a positive constant.

To construct solutions of the equation (1) can be used the following formulae:

$$z_{n+1} = \frac{\lambda_{(n)}}{\lambda_{(n)} - \lambda} u_{n+1} + \sum_{k=0}^n u_k,$$

where  $\lambda_{(n)} = \mu_n^{-1}, u_n = (\lambda T)^k u_0$  and  $u_0 = (L - B)^{-1} s$

if  $T = (L - B)^{-1} C$  and  $u_0 = C(L - B)^{-1}$  if  $T = C(L - B)^{-1}$ .

Theorem 2. Let the inequality  $|\lambda| < \lambda_0$  be satisfied. Then there exists one and only one solution  $z$  of the equation (1), where  $s$  is an arbitrary element in  $X$ . Moreover in the norm of  $X$  it holds  $z_{n+1} \rightarrow z$ . The rapidity of the convergence is greater than in the usual method of successive approximations.

Using special functionals  $y'_n, z'_n$  we obtain on the one hand some known iterative methods, on the other hand till now not used iterative processes.